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Using Intentional Mistuning in the Design of Turbomachinery Rotors

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The maximum blade forced response amplitudes for mistuned turbomachinery rotors are generally much greater than those of their tuned counterparts. However, it is known that the ratio of mistuned to tuned maximum vibration amplitudes, the amplitude magnification, is often largest at a relatively small level of mistuning. Increasing the level of mistuning beyond this critical value actually leads to a decrease in the amplitude magnification. This suggests that it might be beneficial to introduce some level of mistuning into the nominal design of the system intentionally. In this study, the effectiveness of this intentional mistuning strategy is investigated. Intentional mistuning is introduced into the rotor design by varying the nominal blade stiffnesses in harmonic and square-wave patterns. In addition, the unavoidable, random mistuning of the blades is included in the model as usual. The statistics of the forced response are examined for lumped parameter models, as well as for a finite element based reduced-order model of an industrial rotor. For the cases considered, it is found that intentional mistuning can greatly reduce a rotor's sensitivity to random mistuning.

Nomenclature

A = amplitude of intentional mistuning
C = engine order of excitation
c = blade viscous damping coefficient
f = external force on a blade
h = harmonic of intentional mistuning
j = $\sqrt{-1}$
$\boldsymbol{K}$ = stiffness matrix
k = blade stiffness or modal stiffness
k_c = coupling stiffness
k_0 = tuned blade stiffness
m = blade mass
N = total number of blades
q = blade displacement amplitude
R = coupling ratio
t = time
x = blade displacement
γ = blade structural damping factor
Δ = intentional mistuning factor
δ = random mistuning factor
ζ = blade viscous damping factor
λ = eigenvalue
σ = standard deviation of the random mistuning factor
ϕ = phase angle
ω = frequency
ω_0 = tuned blade natural frequency
$\bar{\omega}$ = dimensionless frequency

Subscripts

b = blade component modal coordinates
d = disk component modal coordinates
m = mode number
n = blade number

Superscript

i = rotor number, as taken from a population of randomly mistuned rotors

Introduction

IN general, the blades of a turbomachinery rotor are intended to be identical. However, there are always small, random deviations in the blade properties due to factors such as manufacturing tolerances and in-operation wear. These blade-to-blade discrepancies are called mistuning. Mistuning can have a drastic effect on the dynamics of a bladed disk. In particular, it can lead to the vibration response being mostly concentrated in a small region of the structure, a phenomenon known as localization.[1,2] Furthermore, because of the spatial confinement of vibration energy, certain blades in a mistuned system may suffer a significant increase in forced response vibration amplitudes compared to the ideal (tuned) system.[3–5] The attendant increase in blade stresses can lead to premature fatigue of the blades. It is, therefore, of great interest to be able to predict, as well as to reduce, the maximum blade forced response amplitudes.

Note that the variable of interest is the largest forced response amplitude found for any blade in a frequency sweep. One can define an amplitude magnification factor as the ratio of the maximum mistuned forced response amplitude to the maximum tuned forced response amplitude. By normalization with respect to the tuned system, this factor provides a quantitative measure of the detrimental impact of random mistuning.

It has been demonstrated that the amplitude magnification tends to exhibit a peak value with respect to mistuning strength.[6–9] That is, the maximum forced response increases, usually rapidly, with increasing mistuning up to a certain level, but a further increase in mistuning actually results in lower forced response amplitudes. This remarkable effect of random mistuning has been demonstrated and explained in an analytical study,[9] as well as documented in a case study of the industrial rotor considered here.[10]

This peak amplitude phenomenon leads one to wonder whether intentional mistuning could be introduced into the design of a bladed disk, to reduce the adverse effects of random mistuning. This idea has been considered, either directly or indirectly, in a few previous studies. For instance, Ewins[11] discussed the possible advantages of bladed disk designs in which blades are grouped into "packets" of shrouded blades. This type of design introduces a special form of mistuning, and Ewins explored the beneficial effects of "detuning" the response of certain modes. Griffin and Hoosac[12] considered measuring the blade-alone natural frequencies and then placing the blades so that they alternated between those with higher and lower

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frequencies. Thus, an alternate mistuning pattern was achieved, and they observed some reduction in the forced response for a lumped parameter model of a bladed disk. However, they did not consider including intentional mistuning in the design of the blades. Crawley and Hall[13] did consider deliberate mistuning in the blade design, as well as random mistuning, and they even found optimal patterns of intentional mistuning. However, the focus of their work was the aerodynamic stability (flutter) of the rotor.

More recently, Rządkowski[14] investigated the transient nozzle excitation of mistuned bladed disks. By examining several configurations of a set of nominally different blades, Rządkowski found that a random configuration led to the greatest increase in stresses, whereas an n-periodic blade arrangement was the best distribution in terms of minimizing the largest stresses. Yiu and Ewins[15] simulated many randomly mistuned realizations of a simple model of a 36-bladed disk, and they used discrete Fourier transforms to find the harmonic components of the best and worst mistuning patterns. However, they did not include intentional mistuning in the design.

Subsequently, in a series of articles reporting work in progress, Castanier and Pierre[16,17] and Brewer et al.[18] investigated the effect of introducing intentional mistuning into the nominal design of a rotor. Thus, intentional mistuning was considered as a type of design in which the blades are not intended to be all identical. Preliminary investigations into the combined effects of intentional and random mistuning were reported in this previous work. In the present work, the key developments and findings are presented. The main contributions of this paper are as follows:

1) To examine a range of designs, harmonic patterns of intentional mistuning are considered.

2) In addition, square-wave patterns of intentional mistuning, which only require two different blade types, are introduced as a more practical design alternative.

3) For all of the designs, the effect of random mistuning on the statistics of the maximum blade forced response amplitudes are investigated.

4) By the investigation of the free and forced response of rotors with and without intentional mistuning, the governing physical mechanisms are illustrated.

5) The application of intentional mistuning to the design of an industrial rotor is examined.

Note that some possibly important effects are not considered here, such as aerodynamic coupling between blades and boundary conditions between adjacent rotor stages. Nevertheless, it is believed that this work provides an important, fundamental investigation into using intentional mistuning as a design strategy.

Finally, some very recent work in this area is noted. Choi et al.[19] considered the optimization of intentional mistuning patterns to reduce forced response amplitudes. Kenyon and Griffin[20] analyzed the effect of harmonic mistuning on a rotor's sensitivity to small perturbations in the mistuning.

This paper is organized as follows. In the second section, a lumped parameter model that represents a mistuned, 12-blade rotor is considered. By the use of Monte Carlo simulations, the amplitude magnification factor is examined for various designs with and without harmonic intentional mistuning. Also, a more practical implementation of intentional mistuning is explored, using only two different nominal blade designs. In the third section, the underlying mechanisms governing the effectiveness of intentional mistuning are explored. The free and forced response of a 24-blade lumped parameter model are investigated, and it is seen that certain free response results highlight the key effects of intentional mistuning on the system. In the fourth section, a case study is performed using a finite element based reduced-order model of a 29-blade industrial rotor. The effectiveness of designs with intentional mistuning are shown by examining the statistics of the forced response for two different engine orders of excitation. In the fifth section, conclusions from this study are summarized.

Lumped Parameter Model

A cyclic chain of N single-degree-of-freedom (DOF) oscillators is used as a simple model of a turbomachinery rotor. This model is shown in Fig. 1. Each oscillator represents a blade, with mistuning

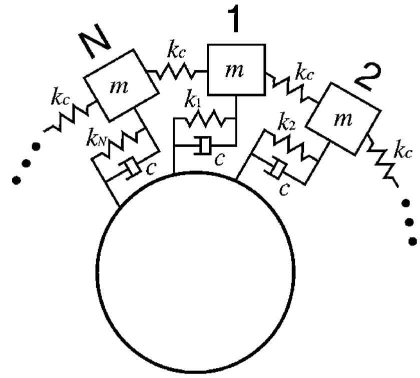


Fig. 1 Lumped parameter model of an N-blade rotor.

modeled as a variation in blade stiffness only; the mass and damping are the same for each blade. Adjacent oscillators are connected by springs of stiffness k_c, which capture the blade-to-blade coupling, for example, structural coupling through the disk or through shrouds.

Although this is an extremely simple model of a rotor, it will be seen that rich vibration phenomena due to mistuning are still captured. Furthermore, the compactness of the system makes it well suited to running parameter studies.

Original Design

For the original, baseline design, all blades are intended to be identical. Thus, the nominal blade stiffness is the same for each blade:

$$k_n = k_0, \qquad n = 1, \ldots, N \tag{1}$$

However, for any actual rotor, some random blade mistuning will be present. For the ith rotor taken from a population of randomly mistuned rotors, the blade stiffnesses are modeled as

$$k_n^i = k_0\left(1 + \delta_n^i\right), \qquad n = 1, \ldots, N \tag{2}$$

where δ_n^i is a dimensionless random mistuning factor. Here, the value of δ_n^i is a sample taken from a random variable with mean zero and standard deviation σ.

The equations of motion for the mistuned system are, thus,

$$m\ddot{x}_n + c\dot{x}_n + k_0\left(1 + \delta_n^i\right)x_n + 2k_c x_n - k_c x_{n-1} - k_c x_{n+1} = f_n \qquad n = 1, \ldots, N \tag{3}$$

In Eq. (3), x_{n-1} and x_{n+1} are aliased to the N blade numbers such that $x_0 \to x_N$ and $x_{N+1} \to x_1$.

The forcing is taken to be engine order excitation, which is harmonic in time and varies only in phase from blade to blade:

$$f_n = f e^{j\phi_n} e^{j\omega t}, \qquad n = 1, \ldots, N \tag{4}$$

The phase at the nth blade is

$$\phi_n = 2\pi C(n-1)/N \tag{5}$$

where C is the engine order.

Furthermore, as a dimensionless measure of interblade coupling strength, the coupling ratio is defined as

$$R = k_c/k_0 \tag{6}$$

A dimensionless frequency is also defined using the natural frequency of a nominal blade:

$$\bar{\omega} = \omega/\omega_0 = \omega\Big/\sqrt{k_0/m} \tag{7}$$

Finally, given harmonic motion such that

$$x_n(t) = q_n e^{j\omega t}, \qquad n = 1, \ldots, N \tag{8}$$

the equations of motion may now be written as

$$\left(-\bar{\omega}^2 + j2\zeta\bar{\omega} + 1 + \delta_n^i + 2R\right)q_n - Rq_{n-1} - Rq_{n+1} = f e^{j\phi_n}\Big/k_0 \qquad n = 1, \ldots, N \tag{9}$$

If one chooses to consider structural damping rather than viscous damping for the blades, then Eq. (9) becomes

$$\left(-\bar{\omega}^2+1+j\gamma+\delta_n^i+2R\right)q_n-Rq_{n-1}-Rq_{n+1}=fe^{j\phi_n}\Big/k_0 \qquad n=1,\ldots,N \quad (10)$$

where γ is the structural damping factor for each blade.

Monte Carlo Simulation

To estimate the statistics of the forced response, a Monte Carlo simulation is performed. The basic procedure is described as follows:

1) Given a value for the standard deviation of random mistuning, the mistuned blade stiffnesses for rotor i are assigned by a pseudo-random number generator. For this 12-blade system, a uniform distribution is used for the mistuning.

2) A frequency sweep is performed to find the largest peak response amplitude of any blade on the rotor.

3) This maximum mistuned response amplitude is divided by the maximum tuned response amplitude to retrieve the amplitude magnification factor for rotor i.

This process is repeated for many realizations of mistuned rotors. By the use of the theory of the statistics of extremes,[21,22] the distribution of the maximum blade forced response will tend to a Weibull distribution. Therefore, the statistics of the amplitude magnification can be estimated by fitting the Monte Carlo results to a Weibull distribution (see Refs. 10 and 16). First, a value is assumed for the location parameter of the Weibull distribution, which is the upper limit of the amplitude magnification. This may be chosen by making a reasonable estimate for the upper bound, or by using an approximation such as that derived by Whitehead.[5] Then, estimates for the other two parameters of the Weibull distribution may be calculated from the data.[22] This allows one to run comprehensive statistical analyses with a relatively small number of Monte Carlo realizations.

To motivate the present study, consider the statistics of the forced response for a 12-blade rotor subject to engine order four excitation. The coupling ratio for this system is taken to be $R=0.02$, and the blade viscous damping ratio is taken as $\zeta=0.001$. For each sampled σ value, the statistics of the forced response were estimated from 500 Monte Carlo realizations with the Weibull location parameter taken to be 2.0.

Figure 2 shows an estimate of the 99th percentile of the amplitude magnification factor as the standard deviation of random mistuning is increased from zero to 8%. The 99th percentile value will be exceeded by only 1% of all mistuned rotors. Recall that the amplitude magnification factor is normalized so that the maximum tuned response corresponds to a value of 1.0, which is marked by the datum line in Fig. 2. Note that as random mistuning is increased to $\sigma=0.02$, the mistuned response becomes as much as 60% higher than that of the tuned system. However, as the random mistuning is increased further, the amplitude magnification decreases to about 36% above the tuned level. This means that, in some cases, tightening the manufacturing tolerances to reduce the blade mistuning can actually lead to an increase in the forced response.

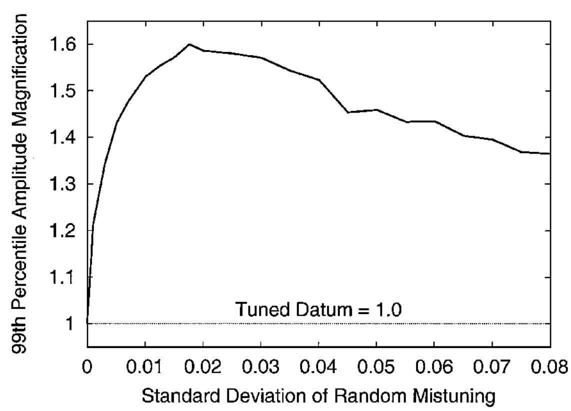


Fig. 2 Amplitude magnification (99th percentile) for the 12-blade system subject to engine order four excitation.

Furthermore, it is seen that for low values of mistuning (1% or so), the response varies greatly with small changes in the mistuning level, indicating that this nominally tuned rotor design has high sensitivity to random mistuning. It would be preferable to have a more robust design that is less susceptible to the harmful effects of random mistuning. Therefore, rather than trying to make all of the blades identical, it might be better to have a design in which the blades are different, to include intentionally some specified pattern of mistuning in the nominal design. Such designs with intentional mistuning are considered next.

Designs with Intentional Mistuning

Intentional mistuning is now included in the design of the rotor. For the original design, each blade has a nominal stiffness k_0. For a design with intentional mistuning, the nominal blade stiffnesses are

$$k_n=k_0(1+\Delta_n), \qquad n=1,\ldots,N \qquad (11)$$

where Δ_n is the intentional mistuning value for blade n.

To consider a range of designs, an approach is adopted that utilizes the cyclic nature of the system. Because any intentional mistuning pattern may be projected onto the harmonics of the system, initial consideration is given only to patterns that are rotationally periodic. This will be referred to as harmonic intentional mistuning. The harmonic intentional mistuning value for blade n is defined as

$$\Delta_n=A\cos[2\pi h(n-1)/N], \qquad 1\le h\le \mathrm{int}[N/2] \qquad (12)$$

where A is the amplitude and h is the integer harmonic of intentional mistuning. Note that the intentional mistuning patterns are restricted to a limited range of harmonics. This is because values of h greater than int$[N/2]$ or less than zero will yield patterns that alias to those in the range [0, int$[N/2]$]. In addition the zero harmonic is neglected because it is equivalent to the original design with stiffer blades, which is a trivial case. Note that because only nonzero harmonics are used in this study, the mean blade stiffness is not changed by the intentional mistuning.

For illustration, a design with harmonic 1 intentional mistuning is shown in Fig. 3. This is referred to as the h1 design, and the other

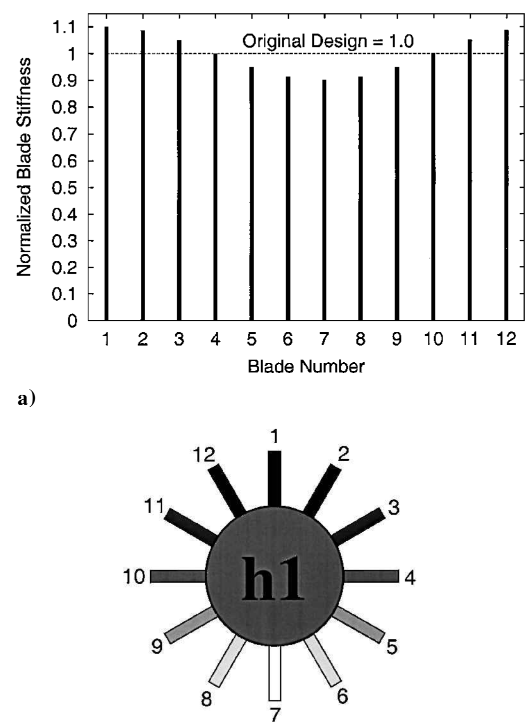


Fig. 3 Design h1: a) normalized blade stiffness values for $A=0.1$ and b) schematic representation.

designs with harmonic intentional mistuning are assigned analogous names.

As a final remark, it is recognized that one could also define a complementary set of harmonic intentional mistuning patterns by using a sine function in Eq. (12). However, this represents only a rotation of a harmonic intentional mistuning pattern. Given that the blade numbering is arbitrary, certain combinations of h and N will result in equivalent designs for the sine and cosine patterns. Furthermore, these will tend to become equivalent designs for each harmonic as the number of blades increases. In any case, for the purposes of this study, it is sufficient to examine a set of single-harmonic patterns of intentional mistuning.

Monte Carlo Simulation for Designs with Intentional Mistuning

Here the effects of random mistuning are considered for rotor designs that include intentional mistuning. For the 12-blade system, there are a total of six designs using the intentional mistuning patterns defined by Eq. (12). These designs are shown schematically in Fig. 4.

Note that even though there is intentional mistuning in the design, superposed on that will be the same random discrepancies in the blade properties (due to manufacturing tolerances, in-operation conditions, etc.) that one would expect for the original design. Thus, for rotor i from a population of randomly mistuned rotors, the blade stiffnesses are modeled as

$$k_n^i = k_0\left(1 + \Delta_n + \delta_n^i\right), \qquad n = 1, \ldots, N \tag{13}$$

With this definition, the random mistuning included in the model is consistent for all designs. It could be argued that the mistuning should scale with the nominal blade stiffness including intentional mistuning, not just with k_0. However, because intentional and random mistuning are both small, this adjustment would be a second-order effect, and it is neglected here.

Figures 5–7 show the 99th percentile of the forced response amplitude magnification vs random mistuning strength for the six designs with intentional mistuning. The original, baseline design is the same as that used for Fig. 2. The amplitude of the intentional mistuning

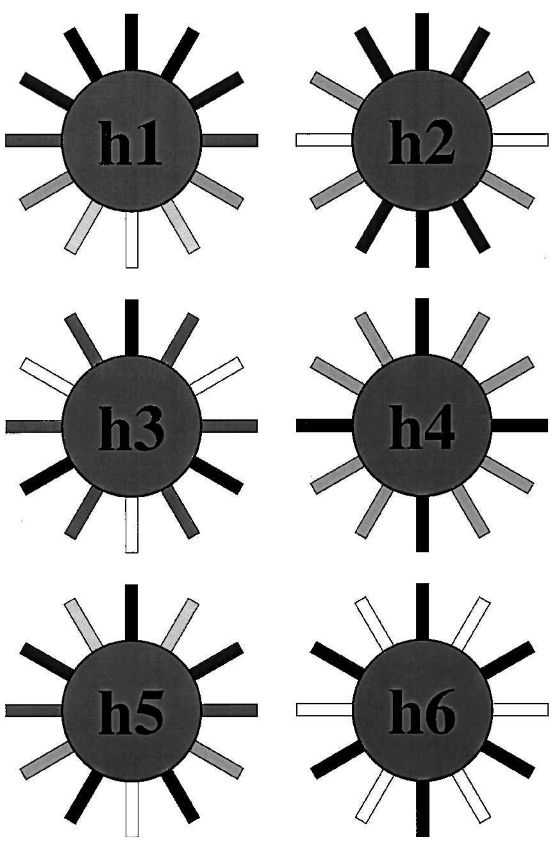


Fig. 4 Schematic representations of six designs with intentional mistuning.

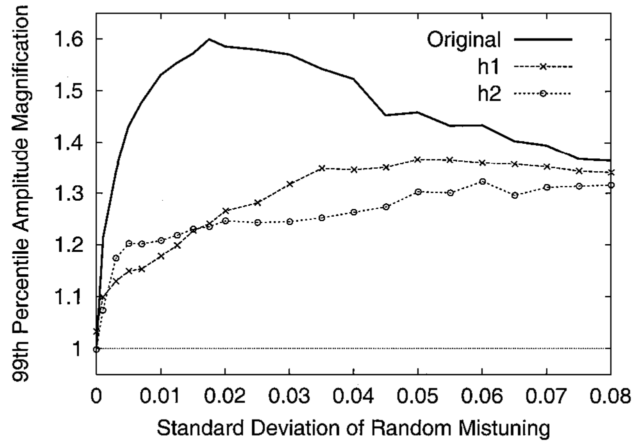


Fig. 5 Amplitude magnification (99th percentile) for the original, h1, and h2 designs.

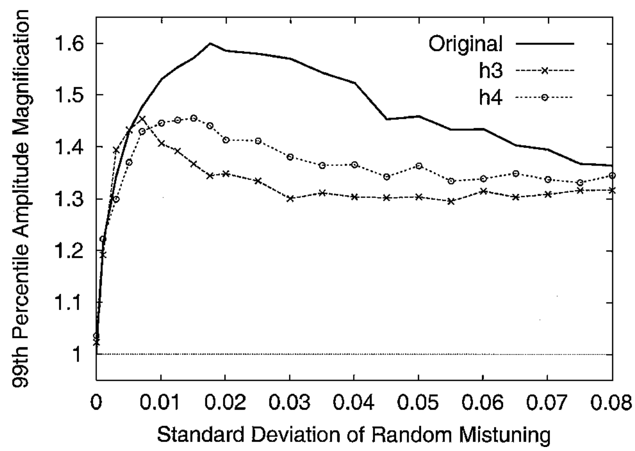


Fig. 6 Amplitude magnification (99th percentile) for the original, h3, and h4 designs.

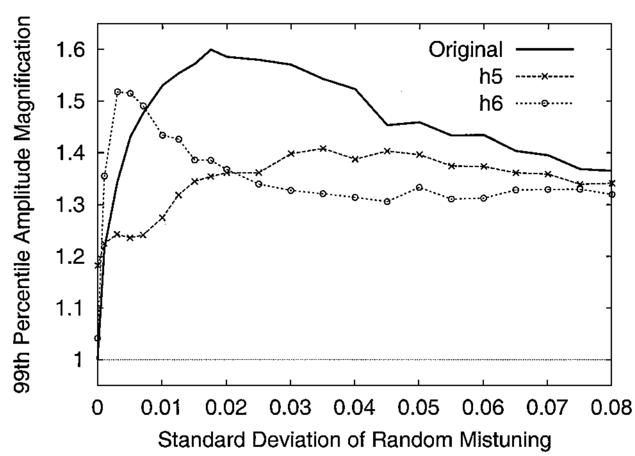


Fig. 7 Amplitude magnification (99th percentile) for the original, h5, and h6 designs.

is $A = 0.1$, which corresponds to approximately $\pm 5\%$ variation in the blade-alone natural frequencies for these designs.

In Fig. 5, note that the h1 and h2 designs result in a dramatic decrease in the amplitude magnification relative to the original design. Of particular interest is that the peak in the magnification around 2% random mistuning is eliminated. This means that tightening the manufacturing tolerances would improve the performance of the system rather than perhaps causing a surprising and discouraging increase in the maximum blade amplitudes.

In Figs. 6 and 7, it is seen that the decrease in amplitude magnification is generally less significant for these designs with higher

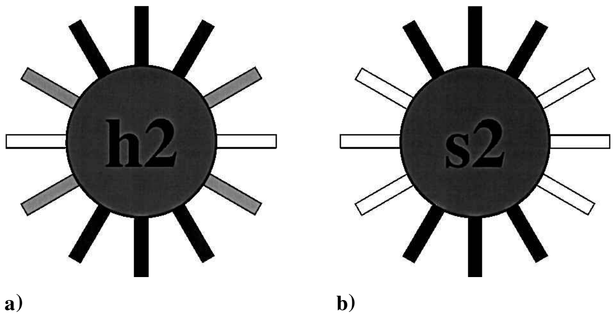


Fig. 8 Comparison of the a) h2 design (four blade types) and the b) s2 design (two blade types).

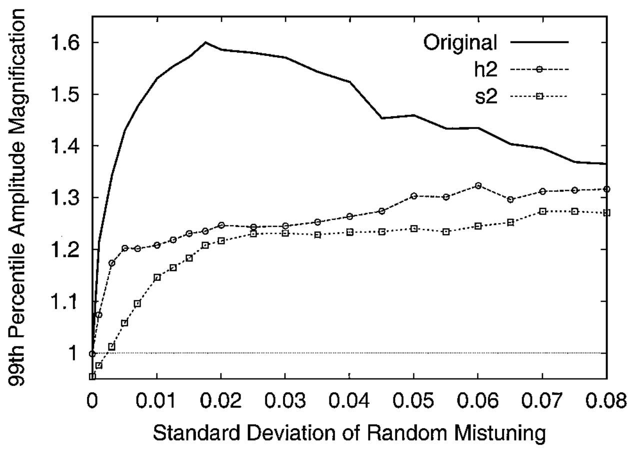


Fig. 9 Amplitude magnification (99th percentile) for the original, h2, and s2 designs.

harmonics of intentional mistuning because the response levels are as high as 40–50% above the tuned reference. In fact, for some low random mistuning values, the h6 design shows a marked increase in the amplitude magnification relative to the original design. Note that the h6 design consists of two different blade types in an alternating configuration. Thus, this nominal design could be considered a tuned six-sector system, which in this case is still susceptible to the effects of random mistuning.

The results for the h1 and h2 designs are encouraging. Figure 5 demonstrates that it is possible to reduce greatly the maximum blade response using intentional mistuning. However, even if a suitable intentional mistuning pattern is found, there are practical concerns that limit the application of intentional mistuning to rotor design. For instance, even for this rotor with only 12 blades, the h2 intentional mistuning pattern requires four unique blade designs, as shown schematically in Fig. 8. Requiring four different blade types for each rotor would likely cause a significant increase in manufacturing costs. It is of interest, then, to determine whether a more practical pattern requiring only two blade designs may be used to obtain similarly good results.

To this end, consider the intentional mistuning pattern shown in Fig. 8b. This is a square-wave approximation of the h2 pattern using only two different nominal blade types, the maximum-stiffness and minimum-stiffness blades, instead of four. This square-wave intentional mistuning pattern is referred to as the s2 design.

Figure 9 compares the 99th percentile of the amplitude magnification for the h2 and s2 designs with $A = 0.1$. Note that the s2 pattern leads to an even greater reduction in the maximum response amplitude than the h2 pattern throughout the random mistuning range. Although this is only an example case for a simple model of a rotor, these results certainly show promise for using intentional blade mistuning to mitigate the damaging effects of random blade mistuning.

Key Mechanisms of Intentional Mistuning

In the preceding section, it was shown that the mistuned forced response can be reduced by including intentional mistuning in the rotor design. In this section, the mechanisms by which the intentional mistuning makes a rotor design more robust with respect to random mistuning are explored. To examine this topic, both the free and forced response are considered for one realization of a mistuned rotor for the original, tuned design and for a design with harmonic intentional mistuning.

As in the preceding section, the lumped parameter model shown in Fig. 1 is used. However, now a 24-blade rotor is considered, and each blade has structural damping rather than viscous damping, with structural damping factor 0.002. The coupling ratio for this system is $R = 0.01$. A random mistuning pattern was assigned by taking samples from a normal distribution with standard deviation $\sigma = 0.5\%$. This mistuning pattern was applied to the original design, as well as an h2 design ($h = 2$ and $A = 0.05$).

Figure 10 shows the forced response to an engine order seven excitation for the two mistuned rotors. The blade response amplitudes are shown for excitation frequencies equal to the system natural frequencies. The response pattern is plotted on the same scale for each excitation frequency, and the corresponding eigenvalue ($\lambda = \bar{\omega}^2$) is labeled. Note that because the system has only structural damping, the natural frequencies are equal to the resonant frequencies for the individual modal responses. Furthermore, because the damping is light, the modal overlap is sufficiently low that the peak blade response (of any blade at any frequency) occurs at one of these natural frequencies.

In Fig. 10, the maximum blade response is seen at $\lambda = 1.0283$ for the original design and at $\lambda = 0.98322$ for the h2 design. Note that the maximum response is about 75% higher for the original design than for the h2 design. Thus, the worst-case response is significantly reduced by including intentional mistuning in the design, which is consistent with the results in the preceding section.

However, when all of the forced response patterns shown in Fig. 10 are considered, note that those of the h2 design generally exhibit a greater confinement of the vibration response, more localization, and higher amplitudes. It is only at a few frequencies (e.g., at $\lambda = 1.0271$, 1.0283, and 1.0312) that the response of the original-design rotor exceeds the worst-case response (at $\lambda = 0.98322$) of the h2-design rotor. Still, the worst-case response of the original design (at $\lambda = 1.0283$) is significantly higher than that of the h2 design. To understand these results, the modes of free vibration of the two systems are now examined.

Figure 11 shows all of the mistuned mode shapes for both designs of the 24-blade system. For each mode plot, there is an accompanying bar graph showing the relative magnitudes of the nodal diameter components (0–12) of the mode shape. Note that, for the tuned case, the system has cyclic symmetry, and, thus, the tuned modes are periodic in the circumferential direction. This periodicity leads to nodal lines crossing the disk, and, thus, the tuned system modes are called nodal diameter modes. For the mistuned case shown here, the cyclic symmetry is destroyed, and, therefore, the modes are no longer pure nodal diameter modes, but feature a combination of nodal diameter (harmonic) components. Note that the decomposition of a mode shape into nodal diameter components is analogous to a performing discrete Fourier transform.

For the original design, the mistuned system modes shown in Fig. 11 are only moderately localized, and, thus, many modes have one dominant nodal diameter component. An engine order excitation that corresponds to this large nodal diameter component will greatly excite the mode, causing large responses in blades where the mode is localized. Indeed, for the original design, the worst-case response to engine order seven excitation occurs at $\lambda = 1.0283$ (Fig. 10), which corresponds to a mode shape that has a large nodal diameter seven component (Fig. 11). In contrast, for the h2 design, the mistuned system modes shown in Fig. 11 are extremely localized, and the modes no longer have a dominant nodal diameter component. This means that these modes are less likely to have a large response to any particular engine order excitation.

Hence, adding harmonic intentional mistuning to the randomly mistuned system does not decrease the degree of mode localization. In fact, the modes become even more highly localized. However, by increasing the localization and spreading out the nodal diameter content of the modes, intentional mistuning makes the rotor less susceptible to being strongly excited by engine order excitation.

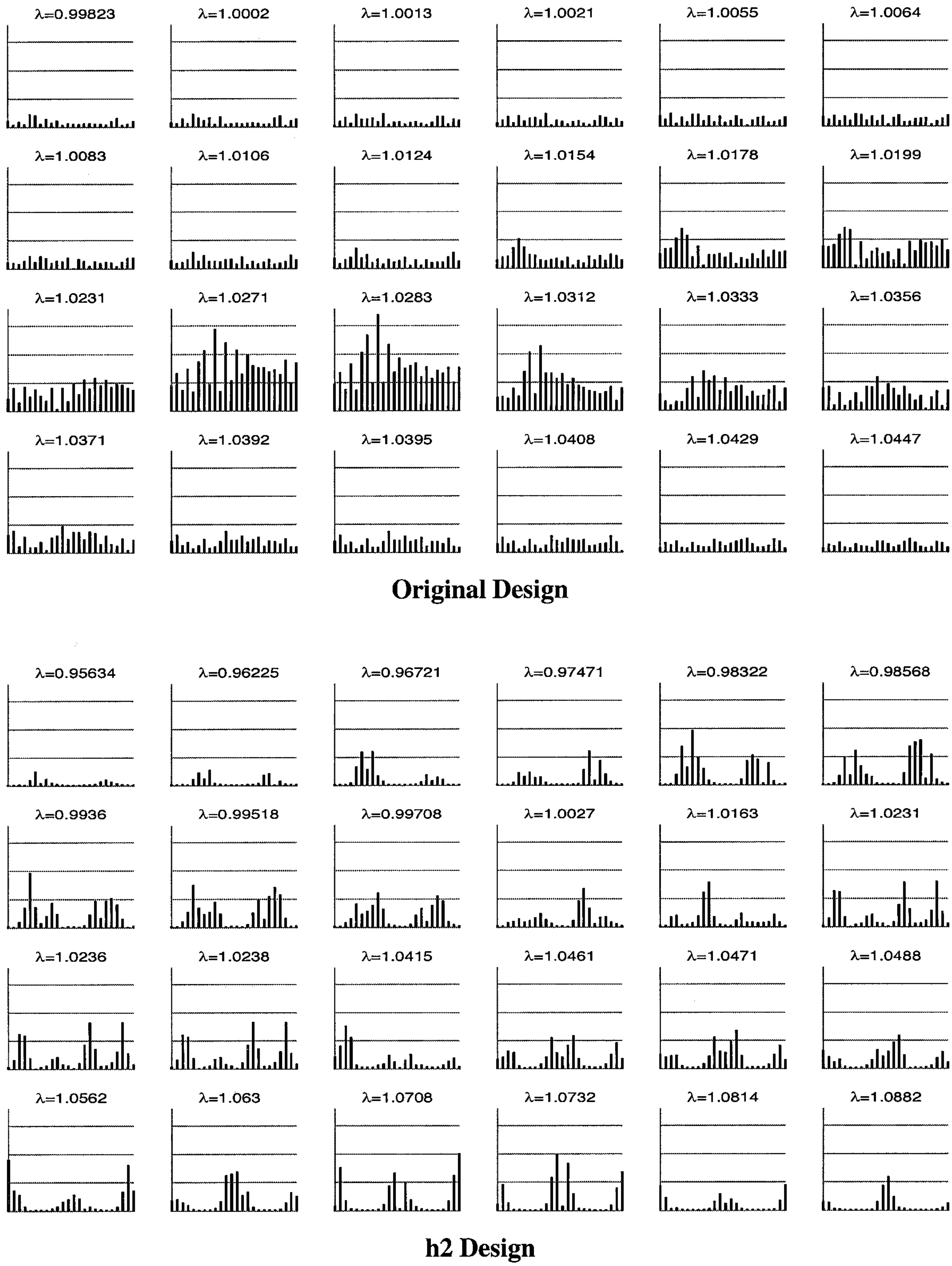


Fig. 10 Forced response ($C = 7$) blade amplitudes of the 24-blade system at excitation frequencies equal to the system natural frequencies ($\lambda = \bar{\omega}^2$) for the original and h2 designs with the same random mistuning pattern.

Application: Industrial Rotor

Thus far, the intentional mistuning concept has been demonstrated with simple lumped parameter models. In this section, the application of this design strategy to a 29-blade compressor stage of an industrial gas-turbine engine is considered.

The finite element mesh for this rotor is shown in Fig. 12. When eight-node solid elements are used, the finite element model (FEM) of the full rotor has 126,846 DOF. Ideally, one could use this FEM in a Monte Carlo simulation to calculate the statistics of the forced response vibration amplitudes. However, the size of this FEM makes such an analysis prohibitively expensive.

Therefore, a previously developed reduced-order modeling technique[23–25] is used to generate a much smaller model of this rotor. This technique employs a component mode approach. One component structure is a blade fixed at the root, or a cantilevered blade. The other component structure is a disk with massless blades attached; for convenience, this is referred to as the disk component. The modes of the disk component are basically those of the disk alone, with the massless blades following the motion of the disk. By the use of these disk and blade component mode shapes to describe the motion of the full bladed disk, the equations of motion for the reduced-order model (ROM) can be derived.[25]

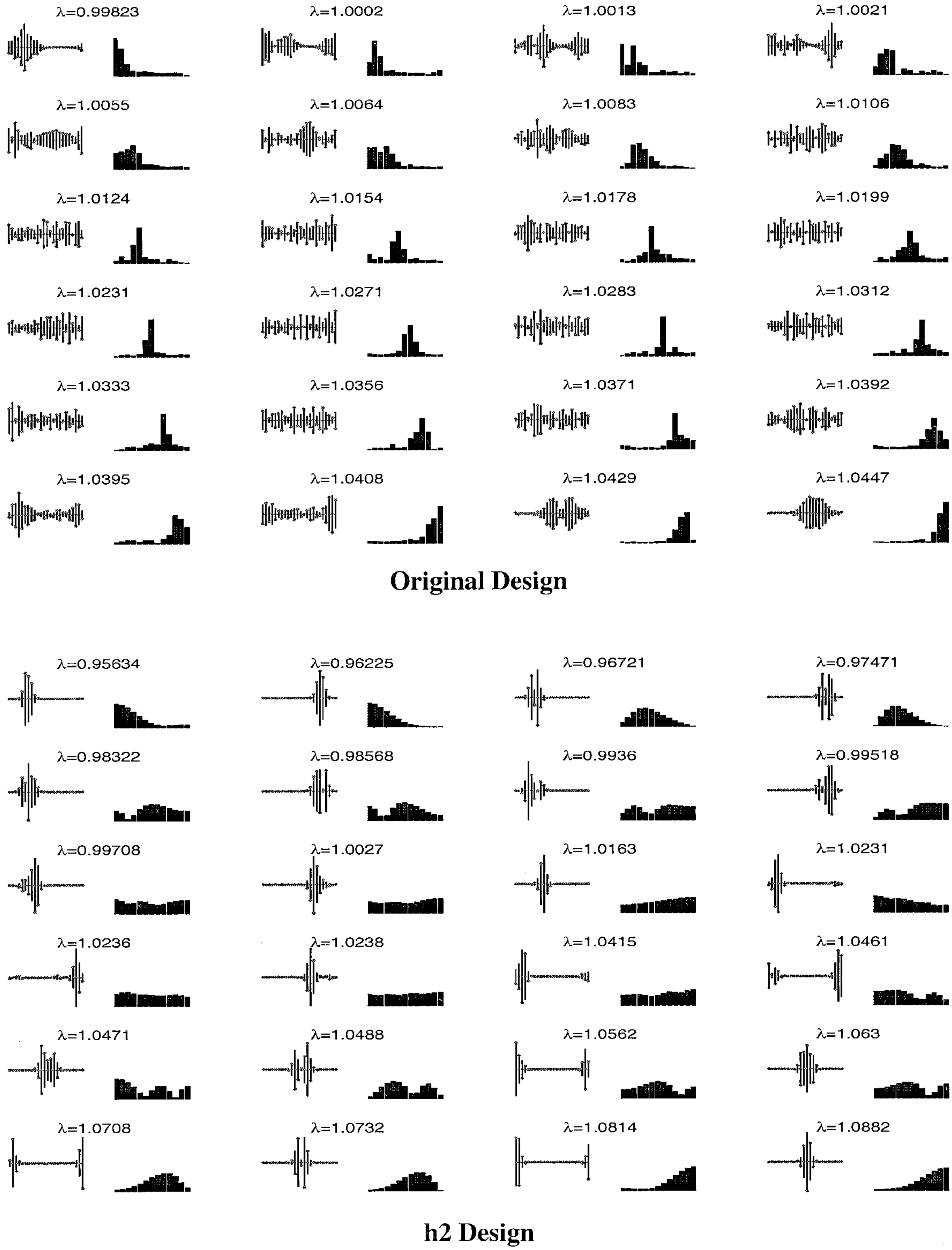


Fig. 11 Modes (on left for each subplot) and nodal diameter components (bold bar graph on right for each subplot) of the 24-blade system for the original and h2 designs with the same random mistuning pattern.

The number of DOF for the ROM depends on the number of component modes selected. For this study, five cantilevered blade modes were selected. Because there are 29 blades, the five families of cantilevered blade modes yield 145 DOF for the full rotor. There were also five families of disk component modes selected, which also yields 145 DOF due to the full set of nodal diameter modes included for each family. Thus, the ROM has a total of 290 DOF. The reduced-order modeling of this rotor is covered in detail in the work by Bladh et al.[10]

Note that improved reduced-order modeling techniques[26,27] have been developed recently that provide significant increases in efficiency and accuracy. However, for the particular rotor and operating conditions considered here, the reduced-order modeling method used has been shown to be quite accurate relative to the FEM.[10] Therefore, it is more than adequate for the purposes of this study.

Amplitude Magnification for the Original Design

The equations of motion for the ROM will not be rederived here. However, to demonstrate how mistuning is modeled, the ROM stiffness matrix is briefly considered. The stiffness matrix for the ROM is of the form

$$K = \begin{bmatrix} K_d & K_{\mathrm{db}} \\ K_{\mathrm{db}}^T & K_b \end{bmatrix} \tag{14}$$

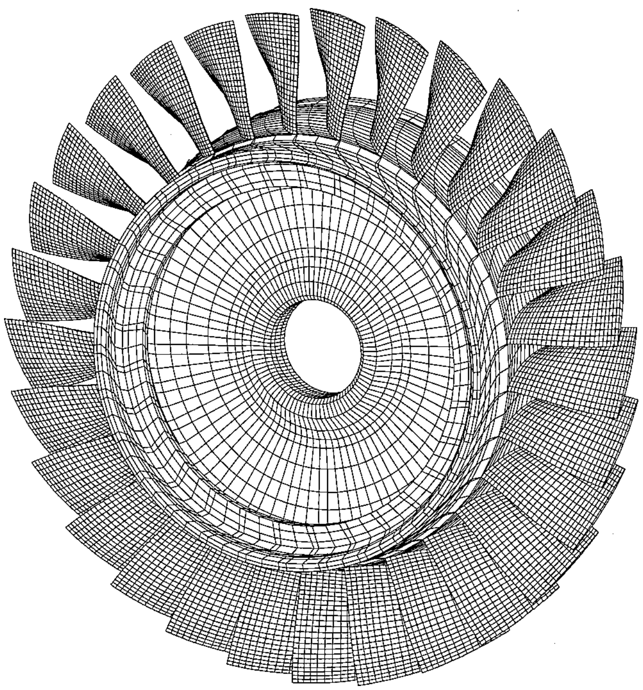


Fig. 12 Finite element mesh for the 29-blade industrial rotor.

where K_d is a diagonal matrix of modal stiffnesses for the disk component modes, K_{db} is a matrix of disk–blade coupling terms, and K_b is a diagonal matrix of modal stiffnesses for the cantilevered blade modes.

Assume that k_m is the modal stiffness for the mth cantilevered blade mode. Then, for a perfectly tuned rotor, K_b is of the form

$$K_b = \text{diag}(\underbrace{k_1, k_2, \ldots, k_M}_{\text{blade 1}}, \underbrace{k_1, k_2, \ldots, k_M}_{\text{blade 2}}, k_1, k_2, \ldots) \tag{15}$$

where diag( ) denotes a diagonal matrix with the argument being the diagonal elements, and M is the number of cantilevered blade modes selected. For this industrial rotor, random mistuning is modeled by introducing small variations in the modal stiffnesses of the blades. Thus, each blade has a slightly different natural frequency for a given mode type.

Consider an arbitrary ith rotor from a population of randomly mistuned rotors having the same nominal design. The mth modal stiffness for blade n of mistuned rotor i can be written as

$$k^i_{mn} = k_m\left(1 + \delta^i_n\right) \tag{16}$$

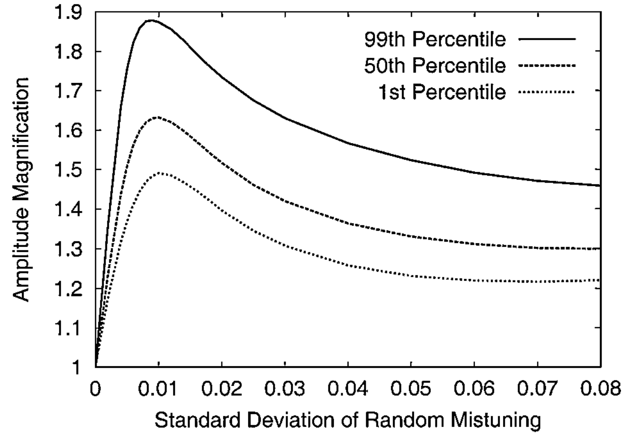
where δ^i_n is the random mistuning factor. For this industrial rotor, δ^i_n is taken from a uniformly distributed random variable with zero mean. For each sampled value of random mistuning strength, 50 Monte Carlo realizations are run, and a Weibull distribution fit is performed using a location parameter value of 2.4. The frequency range of the excitation includes the family of blade second flexural modes. The damping in the model is structural damping, with a structural damping factor of 0.006.

The 99th, 50th (median), and 1st percentile values of the amplitude magnification factor are shown vs random mistuning strength in Fig. 13 for two different engine order excitations. For both an engine order one ($C = 1$) excitation and an engine order eight ($C = 8$) excitation, the 99th percentile shows a peak near 1% standard deviation of random mistuning. This peak is especially dramatic for the engine order one excitation.

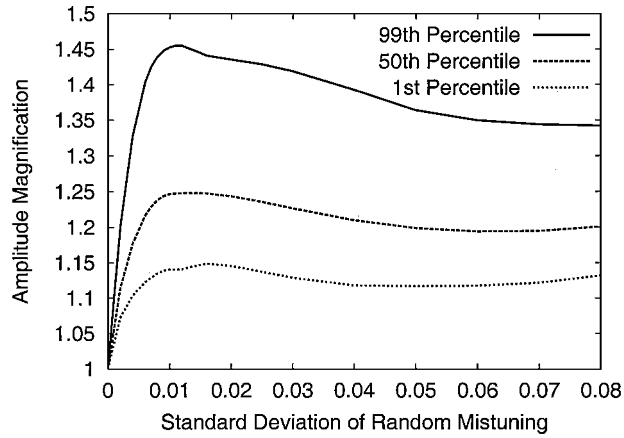
Amplitude Magnification for Designs with Intentional Mistuning

Intentional mistuning is now included in the nominal design of the rotor. For rotor i from a population of randomly mistuned rotors, the mth modal stiffness for blade n is

$$k^i_{mn} = k_m\left(1 + \Delta_n + \delta^i_n\right) \tag{17}$$



$C = 1$



$C = 8$

Fig. 13 Statistics of the amplitude magnification for the industrial rotor subject to engine order excitation.

where Δ_n is the harmonic intentional mistuning value, as defined in Eq. (12).

Figure 14 shows the effects of random mistuning on the 99th percentile amplitude magnification ($C = 1$) for the original design as well as for designs that incorporate harmonic intentional mistuning with $A = 0.1$. Clearly, for these designs, the intentional mistuning leads to significant reductions in the amplitude magnification caused by random mistuning. For the h1 and h2 designs, the intentional mistuning is not as effective in reducing the forced response, although the peak amplitude for each of these designs is still much lower than that for the original design. However, for harmonics three and higher, the designs with intentional mistuning are all extremely effective. Although the peak response for the original design is almost 90% greater than the tuned level, designs h3–h8 keep the response to less than 45% above tuned. Furthermore, the amplitude magnification for the designs with intentional mistuning is fairly constant throughout the range of random mistuning, indicating that the sensitivity to random mistuning is greatly reduced.

The amplitude magnification for an engine order eight excitation is shown in Fig. 15. Intentional mistuning harmonics 4, 6, and 10 are considered, and again the amplitude of intentional mistuning is $A = 0.1$. Although the h10 design is not effective, both the h4 and h6 designs provide a significant reduction in the 99th percentile of the amplitude magnification over a range of random mistuning values.

Distribution of the Amplitude Magnification

Finally, the probability density function (PDF) of the amplitude magnification factor is examined for the case of engine order one excitation. The PDF for the original rotor, no intentional mistuning, is shown for various levels of random mistuning in Fig. 16. As random mistuning is increased from 0.2 to 1%, the distribution spreads and shifts to higher values. As the random mistuning is increased

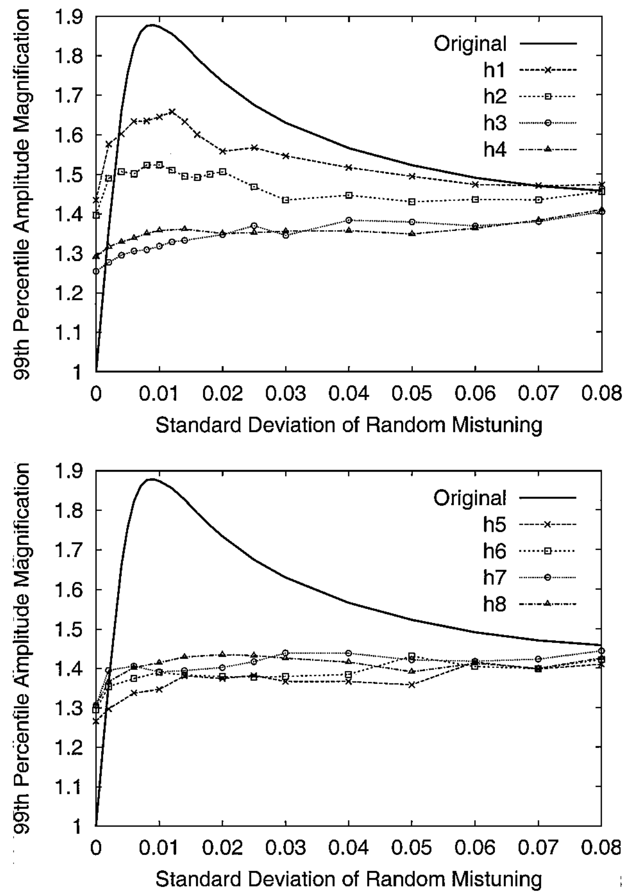


Fig. 14 Amplitude magnification (99th percentile, $C = 1$) for the original design compared to designs with intentional mistuning.

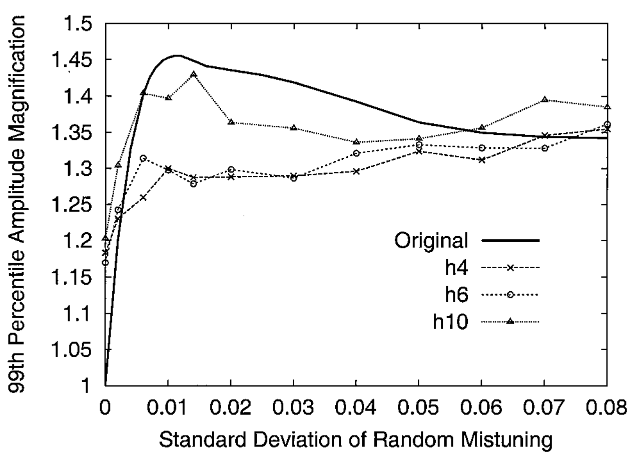


Fig. 15 Amplitude magnification (99th percentile, $C = 8$) for the original design compared to designs with intentional mistuning.

beyond 1%, the trend reverses: The PDF narrows and shifts to lower values. Thus, the sensitivity to mistuning decreases as the random mistuning is increased beyond the critical value.

The PDF of the amplitude magnification for the h4 design is also shown in Fig. 16 for various standard deviations of random mistuning. When these results are compared to those of the original, nominally tuned rotor, it is observed that the design with intentional mistuning not only shows a lower amplitude magnification, but the distribution also has a much narrower range. Note that as the standard deviation of random mistuning is increased from 1 to 4%, the PDF is almost unchanged.

Overall, it is clear that intentional mistuning can greatly reduce the rotor's sensitivity to random mistuning. In particular, significant benefits are seen for the range of random mistuning in which large amplitude magnification is observed for the nominally tuned design.

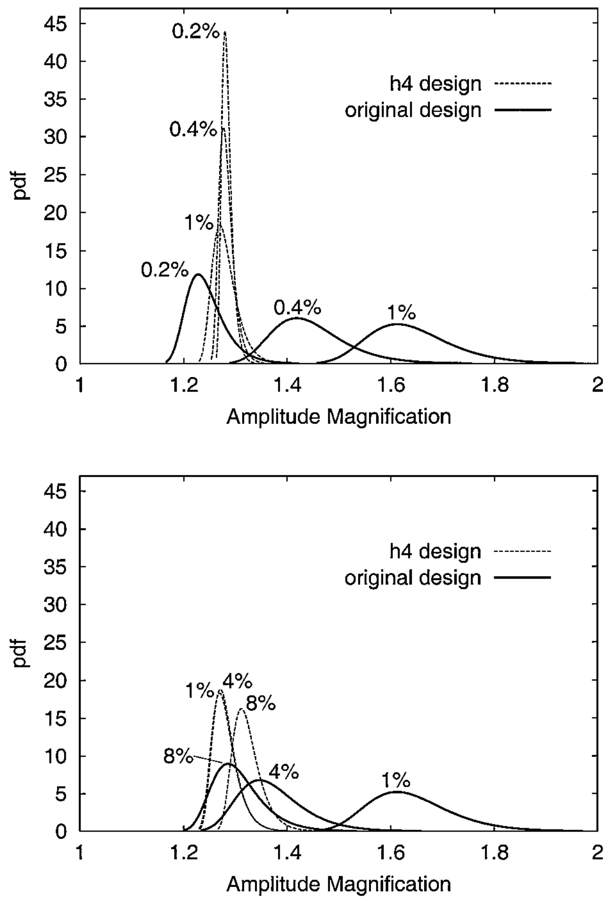


Fig. 16 PDF of the amplitude magnification ($C = 1$) for the original and h4 designs, shown for various standard deviations of random mistuning.

Conclusions

The use of intentional mistuning in the nominal design of a turbomachinery rotor was investigated. It is known that random mistuning can lead to a large amplitude magnification factor, which is the ratio of the maximum blade forced response amplitude for a mistuned rotor to that for a tuned rotor. In this study, the effectiveness of intentional mistuning in reducing the rotor's sensitivity to random mistuning was examined.

First, a simple model of a 12-blade rotor was examined to investigate the effect of intentional mistuning on the statistics of the maximum forced response amplitudes. Intentional mistuning was introduced into the model in harmonic patterns. For this rotor, it was found that designs with harmonic one (h1) and two (h2) intentional mistuning resulted in a large decrease in the maximum forced response amplitude relative to the original, tuned design. In addition, a more practical design was considered, which is similar to the h2 configuration but requires only two different nominal blade types rather than four. This design resulted in an even greater decrease in maximum response amplitude than that of the h2 pattern.

Second, a single random mistuning pattern was considered for the original and h2 designs of a 24-blade rotor. Both the forced response for engine order seven excitation and the free response were calculated. For the original design, it was found that the worst-case response occurs at a resonant frequency for which the corresponding mode 1) exhibits some moderate degree of localization and 2) has a strong nodal diameter component that matches the engine order of the forcing. By the addition of intentional mistuning to the design, the dominant nodal diameter components of the modes were eliminated, making the system less susceptible to being strongly excited by engine order excitation.

Third, the effectiveness of intentional mistuning was investigated for a reduced-order model of a 29-blade industrial rotor. For the original rotor design, a large amplitude magnification factor was demonstrated for two cases of engine order excitation. Both cases showed a peak in the forced response at a low value of random mistuning, whereas higher values of random mistuning led to a decrease in the maximum forced response. Several different rotor designs featuring intentional mistuning were considered, and the effect of random mistuning on the forced response of these designs was examined. It was found that, for certain harmonic patterns of intentional mistuning, the amplitude magnification factor was much lower for the intentionally mistuned design than for the tuned design. For an engine order one excitation, the peak value of the 99th percentile amplitude magnification was reduced from almost 90% above the tuned response level to less than 45% above tuned. In general, it was found that intentional mistuning made the rotor design much more robust with respect to random mistuning.

The results of this study show promise for implementing intentional mistuning in the design of bladed disks to avoid the large forced response amplitudes and stresses that may be caused by random mistuning. Note that some possibly important effects, such as aerodynamic coupling between blades, were not considered in this work. However, it is clear that this subject deserves further investigation.

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